A LOCAL EXPLICATION OF CAUSATION

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In BFS we provide a formal structure and a generalization of Dowe's proposal introducing a *generalized explication of 'causation'* (GEC), as we wish to call it.

In our GEC the key role for understanding causation is played by extensive quantities both conserved and non conserved.

In this paper we want to go deeper into the main features of the strictly scientific side of that proposal and to stand out some implications connected with the description of reality in terms of dynamics of continuous media.

In particular the local point of view of the Observer is emphasized.

We adopt conceptual framework , and the consequent language, borrowed from Physics but this does not mean that the use of a physical language will limit the resulting empirical explication only to the field of Physics.

It is the typical language used to describe the continuous variations of quantities, for instance as that adopted in fluid-dynamics but which can be found in all those physical domains where we have to speak in terms of continuity equations.

We shall refer to *causation as a relation* between different Systems.

In this way we include the very large variety of equilibrium and non-equilibrium stationary situations in which a causal connection has to be recognized.

The acknowledgment of a causal connection between systems does not require any a-priori spatial-temporal relation as shown in the discussion and by the examples below.

The acknowledgment of a causal connection between two Systems is equivalent to say that the two Systems are in interaction

Interactions in mechanical systems

When we do not need to differentiate the specific kinds of interaction taking place between the systems, we correctly agree to call all these forces as "newtonian forces or mechanical interactions".

The *name* "force" was introduced by Newton as a mathematical concept to represent mathematically the fluxion of the fluent[1] named "quantity of movement" but it is the latter quantity (and it is an extensive quantity) which plays the central role in his Principia (Newton, 1712).

The possibility of regarding to mechanical interactions in terms of momentum currents between systems is very appealing and deserves some deeper consideration.

The adoption of this perspective for mechanical forces is the basis of our GEC which can be generalized to all phenomena in the way we are proposing.

This alternative point of view was suggested by Planck (1908), Weil (1924)

In order to have a clear representation of the momentum currents it is convenient to examine the flux lines for each component of the momentum currents because these lines will be represented by the usual three-dimensional lines of flux.

Herrmann & Schmid (1985) discussed the case of electromagnetic interaction

The Maxwell's stress tensor can be written as (Jackson 2001):

$$T_{\alpha\beta} = E_\alpha E_\beta + B_\alpha B_\beta - \tfrac{1}{2} (\mathbf{E}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{B}) \delta_{\alpha\beta},$$

where:

$$dP_\alpha = T_{\alpha\beta} n_\beta d\kappa ,$$

is the flow of the α component of momentum through the surface element with area dκ and normal unit vector **n.**

If we consider a volume V surrounded by a closed surface *K,* the total flow of momentum entering the volume V is:

$$C_{P\alpha} = \int_K dP\alpha = \int_K T_{\alpha\beta}\, n_{\beta}\, d\kappa \; .$$

This is equal to the time derivative of the *total* momentum contained within the volume limited by the closed surface *K*.

By total momentum we mean the sum of the component associated with matter (and which is usually called *mechanical* momentum) and the component *associated to the field* whose density can be written as:

$$\mathbf{g} = 1/c^2 \; (\mathbf{E} \mathrm{x} \mathbf{H})$$

The case of gravitational field has been investigated by Heiduck, Herrmann & Schmid (1987).

They examined the momentum current distribution for each component in the case of the Earth-Moon gravitational interaction treated in the weak field, static approximation.

The procedure is similar to the case of e.m. fields and the resulting pictures of the currents of the different components of momentum are shown.

GEC (Theoretical Context, T)

Def_1: System and state of the system

Def_2: *Extensive quantities* $E = \int \rho(x,y,z,t)\,dV$

Def_3: Intensive quantities $I = \partial E/\partial E'$

Th_1: Extensive quantities and the balance equation

$$\frac{dE}{dt} + C_E = \Sigma_E$$

,

where C_E is the outgoing flux of the extensive quantity E, and Σ_E is its production rate within the volume occupied by the system.

Since P^T is an empirical system, knowing, via measurements, the instantiations of $\{P_1, \ldots, P_r\}$ becomes necessary.

The possibility of knowing the above instantiations requires an abstraction by the Observer:

instantiations are the end-point of a complex chain of operations and they have to be realized in a “condition of simultaneity “ (cfr A. Einstein , Zur Elektrodynamik bewegter Körper. Annalen der Physik 17, 1905, pp 891-921).

The required abstraction concerns the necessity of defining the instantiations at one instant at all parts of the volume of the system.

In a *euleriana* (local) point of view this problem is overcome because it reduces to a local measurement in an (infinitely) small volume.

In a *lagrangian* (substantial) point of view the “abstraction of simultaneity of operations” has to be carried out throughout the whole space generated by the system *and requires the previous definition of simultaneity.* In other words it requires the pre-construction of the space-time.

Def$_4$: Conserved quantity

$$\Sigma_E = 0\ .$$

$$\frac{dE}{dt} + C_E = 0$$

$$\Sigma_S = \sum_{i=1}^{n} \frac{dE_i}{dt} \chi_i$$

The "exchange" of an extensive quantity is symmetrical between the two systems.

The direction of the "exchange" is merely conventional since any situation can be depicted as an "exchange" of the opposite quantity in the opposite direction.

Our explication of the ‘causal interaction is given in a more general sense, and *will not imply a time succession,* as can be also intuitively inferred from what has just been said on the symmetry property of the “exchanges”.

Causation: the integral *explicatum*

We are allowed to speak of *integral causal interaction* between P^T and P'^T, in the time interval , *if and only if*:

1. We observe, relatively to an extensive quantity E_i, that $C_{Ei} = - C'_{Ei}$ and $C_{Ei} \neq 0$ in Δx^0;

2. P^T and P'^T form a system G that is closed with respect to the quantity $E_{i.}$

Causation: the local *explicatum*

From a more general point of view we may see that the structure of the global flux C_E may have two different origins: one with respect to the different regions of the boundary surrounding the system and one concerning the possible superposition of different fluxes at the same point.

$$C_E = \int_K \vec{\Gamma}_E \cdot \vec{n} \, d\kappa$$

The second kind of structure is concerned with the possibility of considering the local flux density as the result of the superposition of different flux densities of the extensive quantity E flowing through the same area and

with the possibility of separating the various components.

Now propose the local *explicatum*

By defining the (infinitesimal) flux entering P^T and P'^T we have that **.** Thus, we are able to speak of *local causal interaction* between P^T and P'^T, *if and only if*:

$$\Gamma_E \cdot \boldsymbol{n} \neq 0$$

at one point of one surface selected to separate the two systems.

Causation in equilibrium and non-equilibrium in stationary states

the local *explicatum*

“In base al concetto del sussistere per sé delle cose, precedentemente formato e verificato dall’esperienza, si evince che la cosa rimarrebbe quello che è, se non intervenisse qualcosa d’altro. Di qui la spinta a cercare una causa per ogni mutamento.”(Riemann, Teoria della Conoscenza).

Is it correct to claim for the presence of interactions if the state of the system changes ?

Vice versa if the state does not change in time (stationary sate) is it correct to claim that the system is not in interaction with other systems?

Both in mechanics and in thermodynamics we need to state, by principle, what a “situation of non interaction” is. In the former case we need to define “inertial motions” (in Newtonian language we could say that their fluents have null fluxions);

in the latter we define equilibrium states as those configurations which are realized in isolated systems i.e. in which interactions with external systems are prevented.

Conversely if an inertial motion is observed can we affirm that the system is not in interaction with something else?

The answer is obviously negative but it is worth pointing out that *this conclusion is entirely dependent on the theoretical context T* (both in a substantial or local point of view) adopted by the Observer.

We have proposed what we have called a *generalized explication of 'causation'* (GEC), both in the integral, less complex form and in the local, general form in which both conserved and non-conserved quantities have the same role.

In particular the local account frees the Observer from the constraints required in the integral point of view.

we have shown that a causal connection between systems can be recognized by the Observer according to his level of complexity.

The role of the Observer is fundamental even in the definition of the boundaries of Systems as was discussed by Boniolo, Faraldo and Saggion (2009).

Definition of boundaries and the acknowledgments of causal connections between Systems constitutes the core of the abstraction performed by the Observer in his interaction with external world.