Gamma-hadron discrimination in EAS

a method based on multiscale, lacunarity and artificial neural network analysis

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Science: Image in Action - Erice Apr, 19 2011

The aim is to introduce a separation technique on an event by event basis based on the different topology of the showers. In this work the technique is applied to ARGO-YBJ simulations. ARGO-YBJ experiment studies cosmic rays at an energy threshold of 100 GeV, by means of the detection of small size air showers. This goal is achieved by operating a full coverage array in Tibet.

ARGO-YBJ unique features:

high time resolution coupled to fine space granularity on a large surface with almost full coverage:

- time resolution of 1 ns
- space granularity corresponding to the strip size (7x56 cm^2)
- active surface of $6000m^2$ with $>$ 90% coverage

A detailed picture of the shower front is mandatory

- particle density distribution
- particle arrival time distribution

A three step method

multifractal behaviour, lacunarity, neural network

The method uses the multiscale concept and is based on the analysis of multifractal behaviour and lacunarity of secondary particle distributions together with a properly designed and trained artificial neural network

- analysis of multifractal behaviour by means of wavelet transforms
- analysis of lacunarity in arrival times with gliding box method
- artificial neural network: standard backpropagation

Basic assumptions

The basic assumption is that shower propagation induces multi fractal behaviour of the particle density distribution in the shower front

- the main motivation for this assumption to be made is the self-similarity of the interactions in shower propagation
- to the same extent it is conceivable that the multifractal behaviour of different primaries could be different
- unraveling these difference is the main task of this analysis
- to pursue this goal the detailed description of the shower front both in space and time, provided by ARGO, is a unique tool and his powerfulness has to be fully exploited
- (multi)fractal behaviour has already been suggested as separation technique for separating the cosmic ray primaries and for mass composition studies [Kempa 1994], [Rastegaarzadeh and Samimi, 2001]

Wavelet method and fractal analysis

The number of particles $N(R)$ inside a radius R is computed.
If a scaling law of the form $N(R) \propto R^D$ holds, the distribution has a fractal distribution with dimension D.
Multifractal behaviour can be revealed by studying the scaling laws for different secondary particles at different core distances.

The wavelet transform is a natural tool to study the scaling laws.
It is a linear operator that can be written as:

$$wavelet(scale, t) = scale^{-1/2} \int_{-\infty}^{+\infty} f(x)\psi^* \left(\frac{x - t}{scale} \right) dx$$

The scale of wavelet transforms

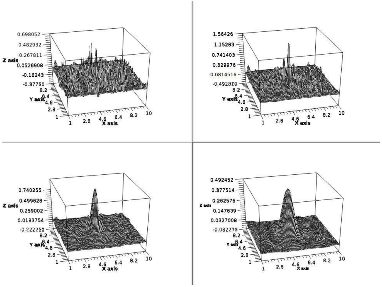
The set of scales are powers of two: $scale = 2^r$. The scale may be considered as the resolution.

In other words, if we perform a calculation on a scale s_0, we expect the wavelet transform to be sensitive to structures with typical size of about s_0 and to find out those structures.

Space pixel $\simeq$ PAD; s=2 $\div$ 32

$\Rightarrow$ R $\simeq$ 1.2 $\div$ 20m

Set of matrices of wavelet coefficients: one matrix for each scale investigated



The fractal dimension

Wavelet transforms can be used as a natural tool to investigate the self-similar properties of fractal objects at different spatial locations and length scales.

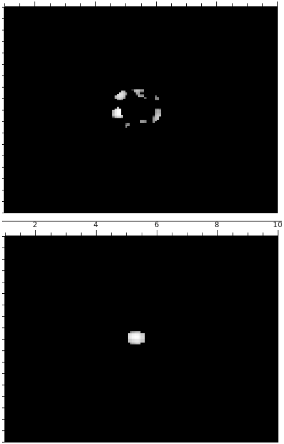
Property: if a function f has scaling law with exponent D around x_0:
$f(R(x_0, \lambda a)) \sim \lambda^D f(R(x_0, a))$
then the wavelet transform
$W(s,t) = \int g(x; s, t) f(x) dx$
has the same scaling exponent,

- The local scaling behaviour is represented by $W(s,t) \sim s^{D(t)}$.
- Therefore for any distribution function f the slope of the plot of $\log W(s,t)$ versus $\log s$ will give the fractal dimension of the distribution around point t for the range of the scale s.

The fractal dimension

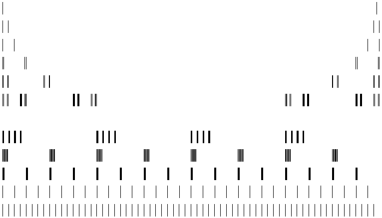
selection of two regions

- We compute a $Log(W)/Log(s)$ matrix for each scale s.
- Selection of two regions: I and O.
- $D = Log(W)/Log(s)$ is the fractal dimension.
- D has a Gaussian distribution: μ, σ.
- Results: $\mu_I, \mu_O, \sigma_I, \sigma_O$



Properties of lacunarity

Lacunarity is a measure of how a fractal fills space. Distribution with the same fractal dimension can show-up differently because of their different degree of homogeneity/space filling structure, i.e. lacunarity. Below: high to low lacunarity, same fractal dimension



Properties of lacunarity

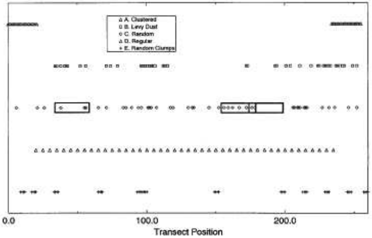
Time lacunarity is computed in the same two regions as the spatial separation. First, we compute time array as $T = T_{max} - T_{min}$ where T_{min} is the time arrival of the first secondary particle (set to 0) and T_{max} is the time arrival of the last one. Then we need to define the time scale on which we compute lacunarity. We call this crucial parameter t_{lac}.

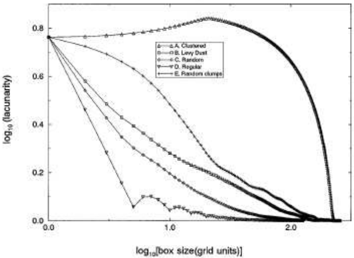
Lacunarity is small when texture is fine and large when texture is coarse.

To measure lacunarity, we adopt the gliding box method (Allain and Cloitre 1991).

Lacunarity

importance of the t_{lac} parameter





Computing lacunarity

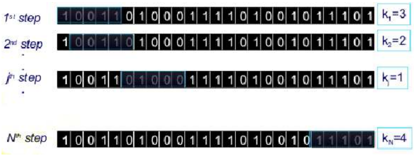
Gliding Box Method

A box of length t_{lac} is placed at the origin of the sets. The number of occupied sites within the box (box mass k) is then determined.

The box is moved one space along the set and the mass is computed again.

This process is repeated over the entire set, producing a frequency distribution of the box masses $n(k, t_{lac})$.

This frequency distribution is converted into a probability distribution $Q(k, t_{lac})$ by dividing by the total number of boxes $N(t_{lac})$ of size t_{lac}.



Computing lacunarity

Frequency distribution is converted into probability distribution:

$$Q(k, t_{lac}) = n(k, t_{lac})/N(t_{lac})$$

First and second moments of the distribution are computed:

$$Z_1(t_{lac}) = \sum_k k \cdot Q(k, t_{lac})$$

$$Z_2(t_{lac}) = \sum_k k^2 \cdot Q(k, t_{lac})$$

The lacunarity is now defined as:

$$\Lambda(t_{lac}) = Z_2/Z_1^2$$

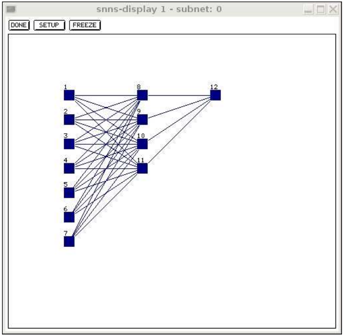
Lacunarity is computed both in the inner and the outer ring (Λ_I, Λ_O). We find that 5 ns is a good choice for the t_{lac} parameter.

Artificial Neural Network

We assume that the mass of the progenitor can be estimated with the use of an artificial neural network of seven variables.

The neural network is a standard three layer perceptron with a hidden layer and one output neuron (1=hadron, 0=gamma). The input layer consist of seven neurons:

- mean of fractal dimensions: μ_I, μ_O
- st. dev. of fractal dimensions: σ_I, σ_O
- time lacunarity: Λ_I, Λ_O
- time array $T = T_{max} - T_{min}$



Testing the method

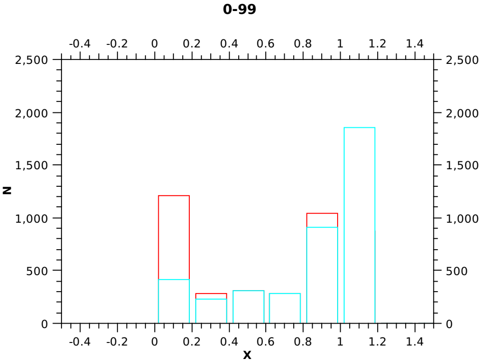
Following results have been obtained working on:

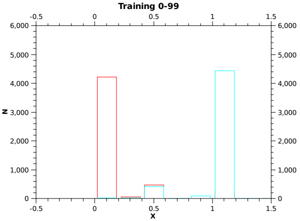
- Simulated showers with zenith angles $0^\circ < \theta < 15^\circ$
- 4 spatial scales ($\approx 2, 4, 8, 16m$)
- Spatial regions: $r < 8m$ and $10 < r < 13m$
- $t_{lac} = 5ns$
- Neural network trained on 50000 cycles
- Learning function is standard backpropagation
- Initial weights randomized

Simulated showers

N. Hits	Photon	Hadron	Events trained	Second set
0-99	>50000	10000	5000 + 5000	5000 + 5000
100-499	15000	1500	1000 + 1000	450 + 450
500-999	1100	228	100 + 100	128 + 128
1000-4999	525	110	50 + 50	60 + 60
5000+	25	5	-	-

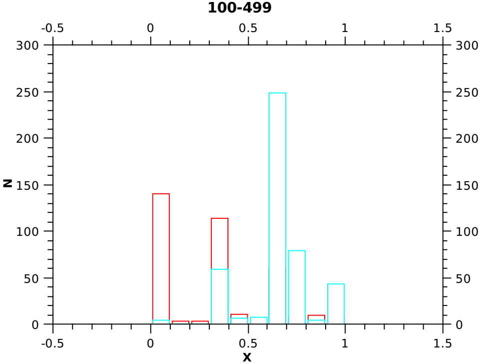
Results on second set and training set
0-99 hits

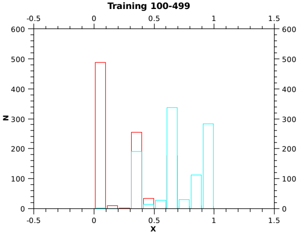




Results on second set and training set

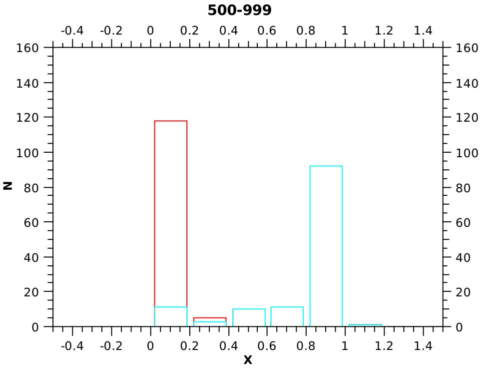
100-499 hits

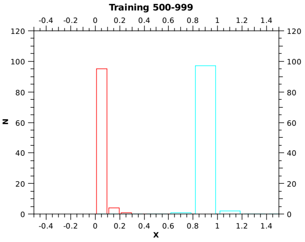




Results on second set and training set

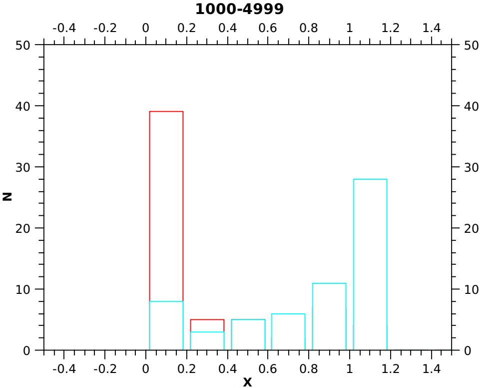
500-999 hits

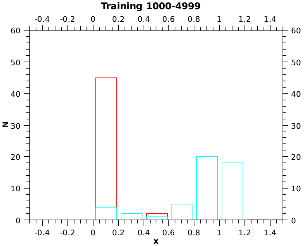




Results on second set and training set

1000-4999 hits





Q values

N. Hits	N. Trained	Q trained set	Q second set
0-99	5000	3.13	0.80
100-499	1000	1.80	1.73
500-999	100	∞	2.80
1000-4999	50	2.83	1.81

$Q = \frac{\varepsilon_\gamma}{\sqrt{1-\varepsilon_h}}$ where ε_γ is the fraction of showers induced by photons correctly identified and ε_h is the fraction of showers induced by protons correctly identified, so that $1-\varepsilon_h$ is the background contamination (fraction of events induced by protons and erroneously identified as gammas)