

ANTI-PHASE SPIKING PATTERNS

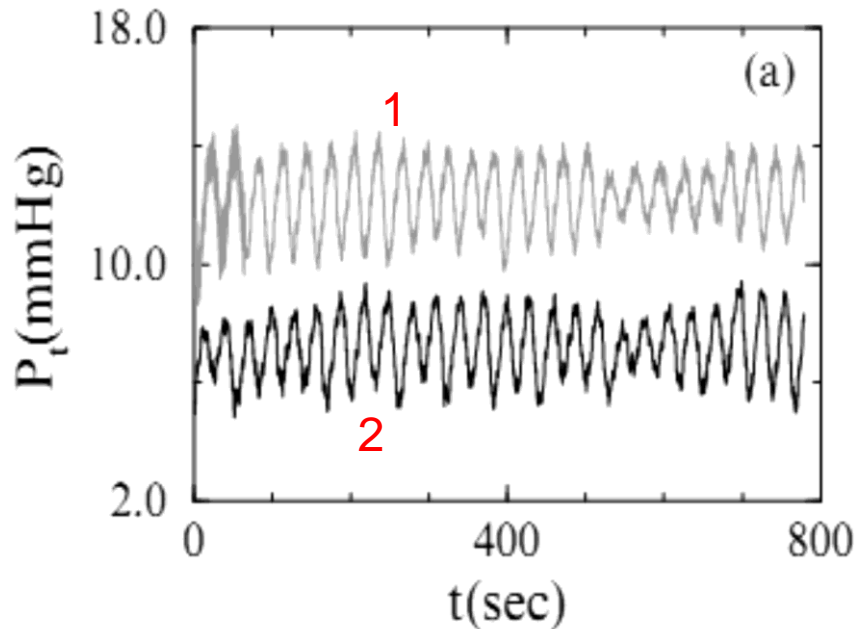
in a Circular Chain of Morris – Lecar Neurons

M. P. Igaev, A. S. Dmitrichev, V. I. Nekorkin

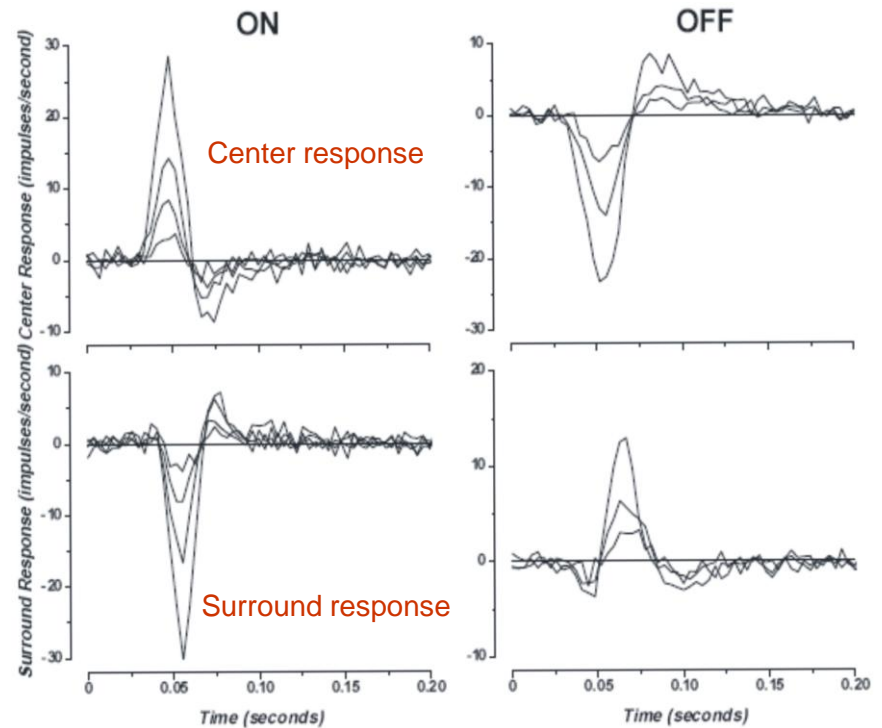
Institute of Applied Physics, Russian Academy of Science

- **Anti-phase synchronization in biological systems**
- **Morris – Lecar neuron model. Circular chain**
- **Anti-phase spiking patterns (ASP)**
- **Properties of the ASP. Multistability**

Examples of anti-phase synchronization in biological systems



Examples of the tubular pressure variation that one can observe in adjacent nephrons (O.V. Sosnovtseva, A.N. Pavlov, E. Mosekilde, N.-H. Holstein-Rathlou, *Synchronization phenomena in multimode dynamics of coupled nephrons*)



The typical responses of P-type primate retinal ganglion cells (ON and OFF), center and surround (E. Kaplan and E. Benardete, *Progress in Brain Research*, 2001)

Morris – Lecar neuron model

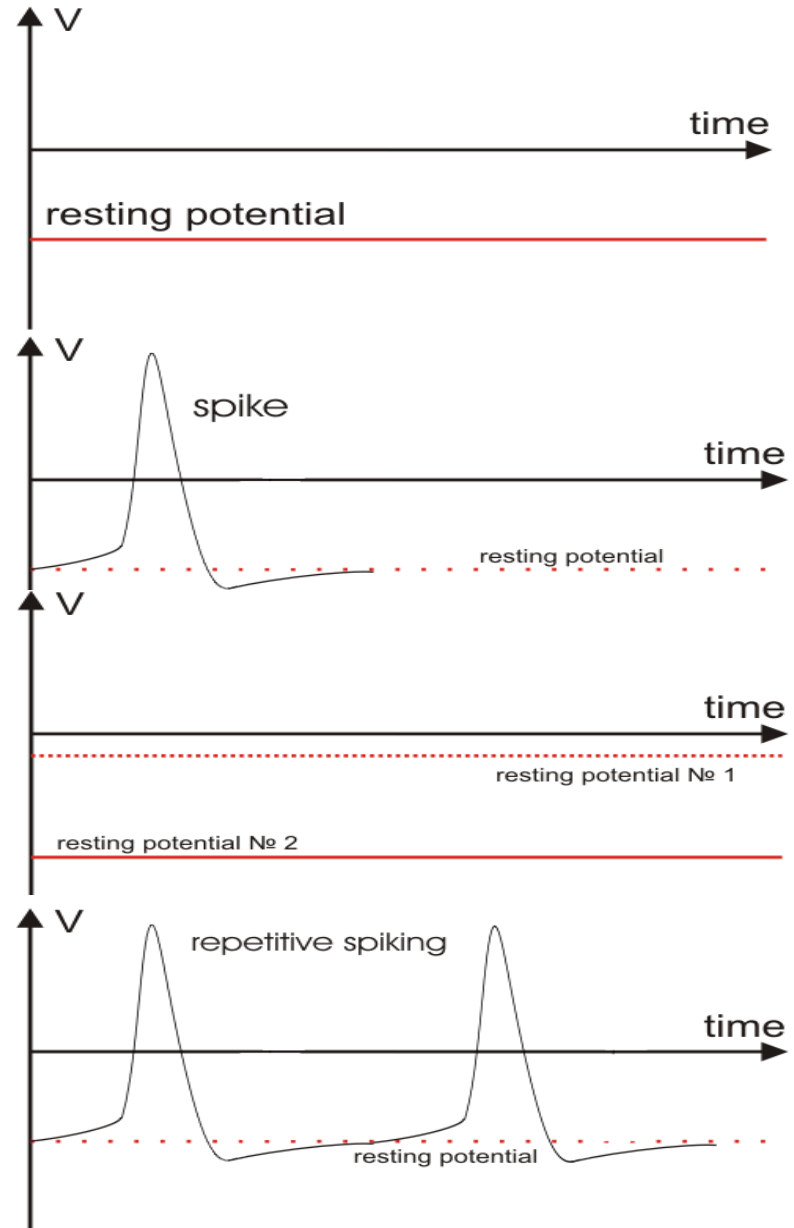
- Morris – Lecar neuron model, suggested as a model for describing oscillatory voltage patterns of barnacle muscle fibers, is represented by:

$$\begin{cases} I_{ext} = C_M \frac{dV}{dt} + I_L(v) + I_{Ca}(V, M_\infty) + I_K(V, n), \\ \frac{dn}{dt} = \frac{N_\infty(V) - n}{\tau_n(V)}, \end{cases}$$

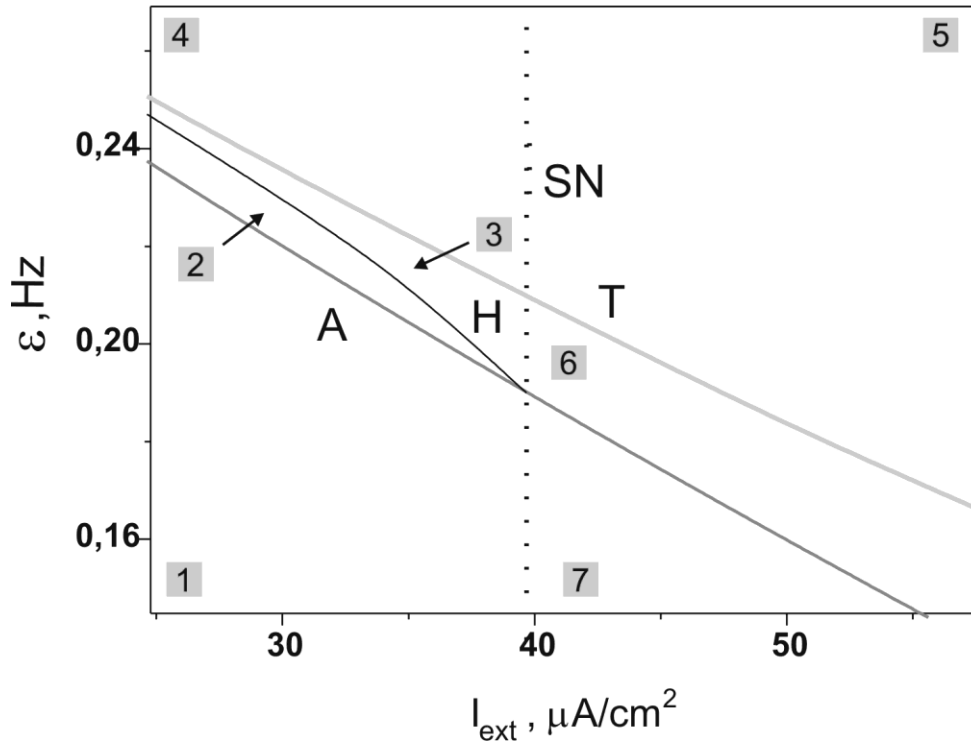
where v – membrane potential of the neuron cell,
 n – activation variable for K^+ - ionic channel

C. Morris, H. Lecar,
Biophysical J., vol.35 (1981)

- Repetitive spiking appears with almost zero frequency. In terms of nonlinear dynamics this follows from saddle-node bifurcation on an invariant cycle.



Morris – Lecar neuron model



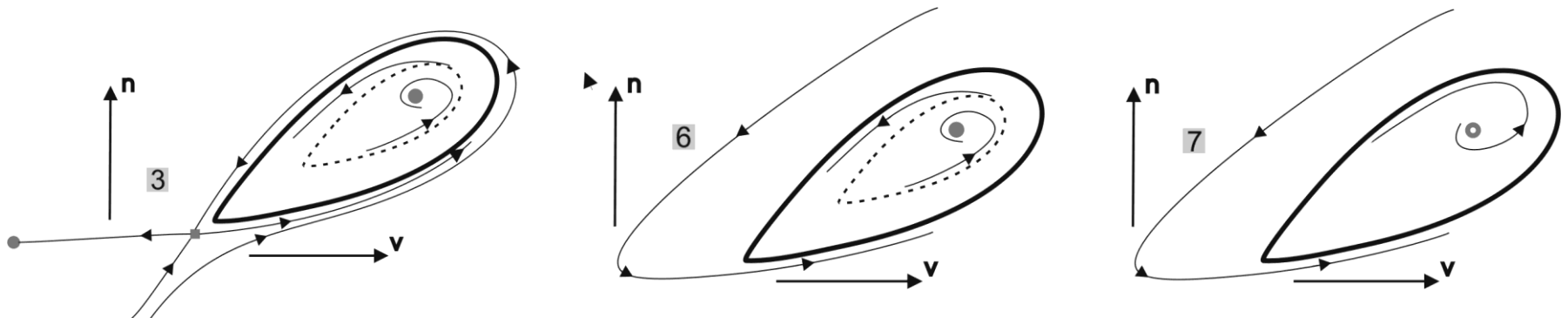
A – Andronov – Hopf bifurcation

H – Homoclinic loop bifurcation

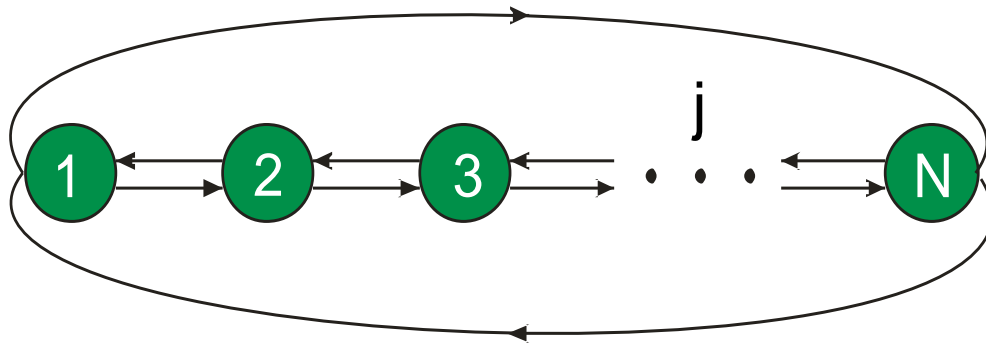
T – Tangent limit cycle bifurcation

SN – Saddle – node bifurcation

K. Tsumoto, T. Yoshinaga and H. Kawakami
*International Symposium on Nonlinear Theory
and its Application (2002)*



Circular Chain of Morris – Lecar Neurons



C. Morris, H. Lecar,
Biophysical J., vol.35 (1981)

$$\left\{ \begin{array}{l} I_{ext} = C_M \frac{dV_j}{dt} + I_L(V_j) + I_{Ca}(V_j, M_\infty) + I_K(V_j, n_j) + d(V_{j-1} - 2V_j + V_{j+1}) \\ \frac{dn_j}{dt} = \frac{N_\infty(V_j) - n_j}{\tau_n(V_j)} \end{array} \right.$$

Weak coupling

$$d \ll \min \{g_L, g_K, g_{Ca}\}$$

$$I_L(V) = g_L(V - V_L)$$

$$I_{Ca}(V) = g_{Ca}(V - V_{Ca})M_\infty$$

$$I_K(V, n) = g_K(V - V_K)n$$

$$N_\infty = 0.5(1 + th \frac{V - V_3}{V_4})$$

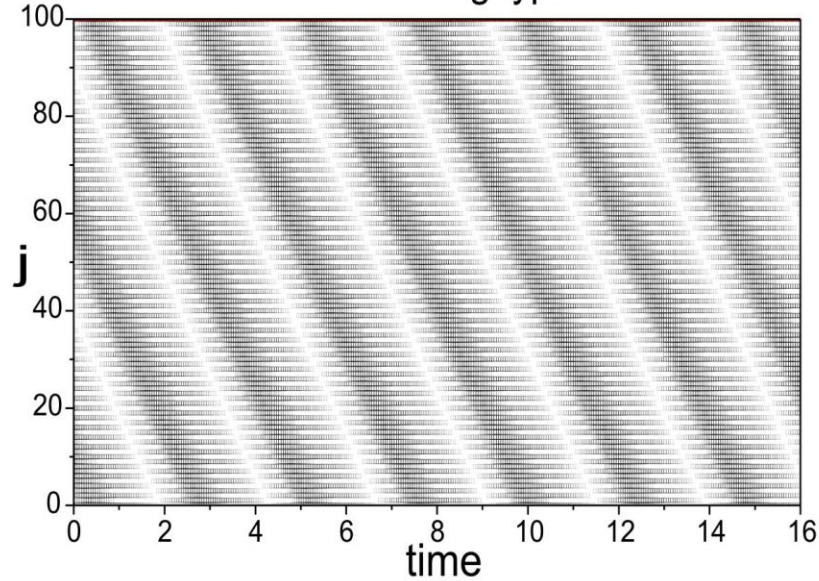
$$M_\infty = 0.5(1 + th \frac{V - V_1}{V_2})$$

$$\tau_n = \frac{1}{\varepsilon} ch^{-1}(\frac{V - V_3}{2V_4})$$

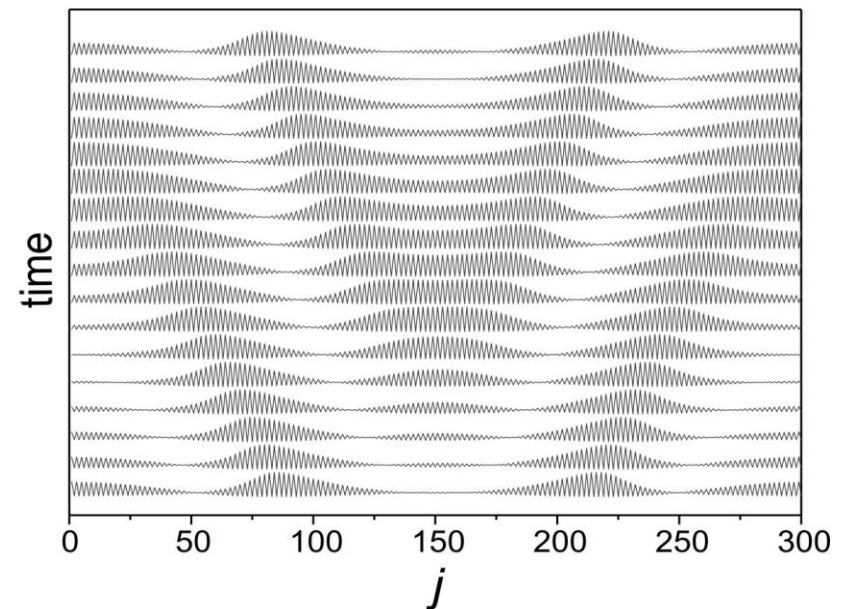
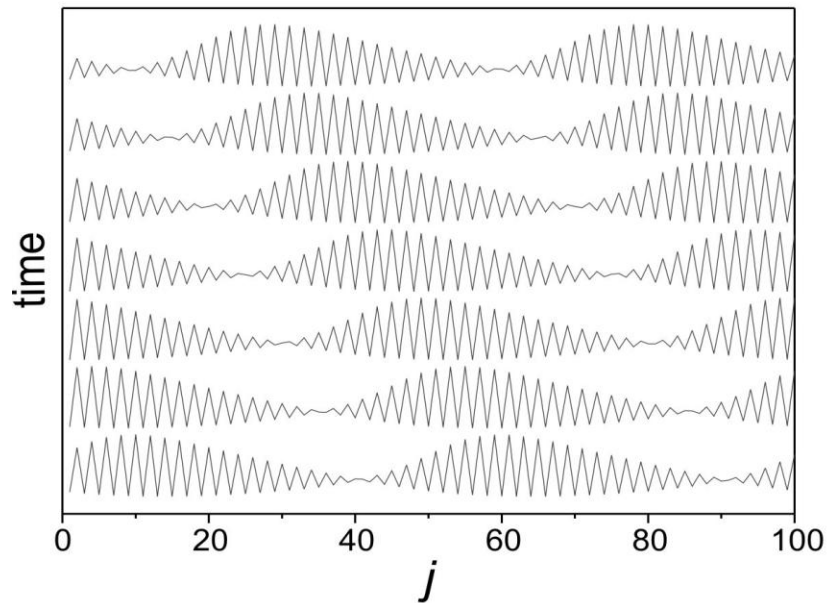
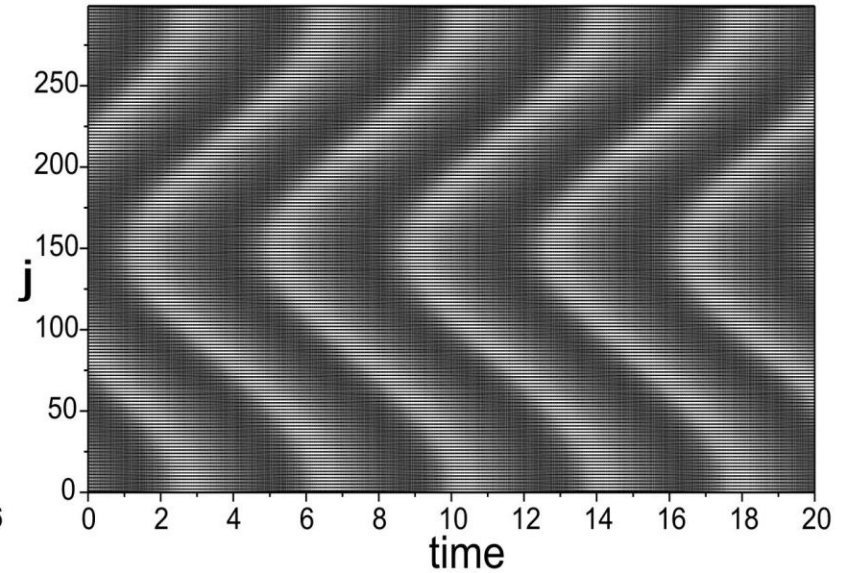
Typical values: $g_L = 2 \frac{mS}{cm^2}$ $g_{Ca} = 4 \frac{mS}{cm^2}$ $g_K = 8 \frac{mS}{cm^2}$ $d = 0,08 \frac{mS}{cm^2}$

Anti-phase spiking patterns (ASP)

Travelling type

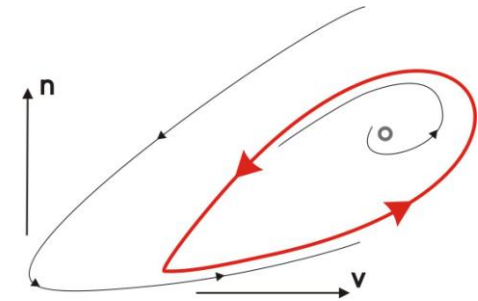


Oscillatory type



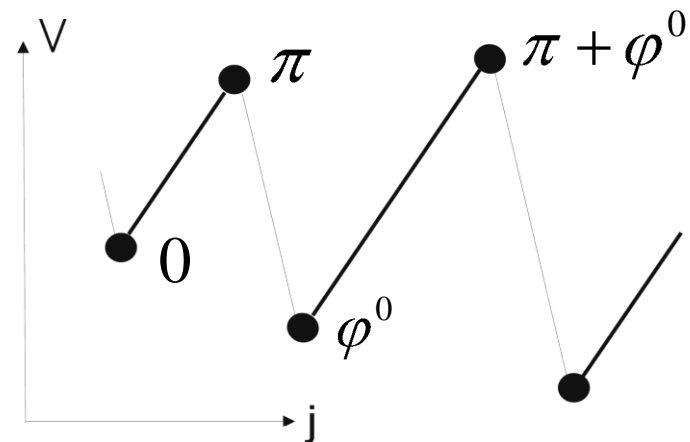
Three requirements for the ASP existence

- Only repetitive spiking regimes of a single neuron are important. Stable **limit cycle attractor** on the phase plane (v, n) is necessary but not sufficient condition for ASP existence

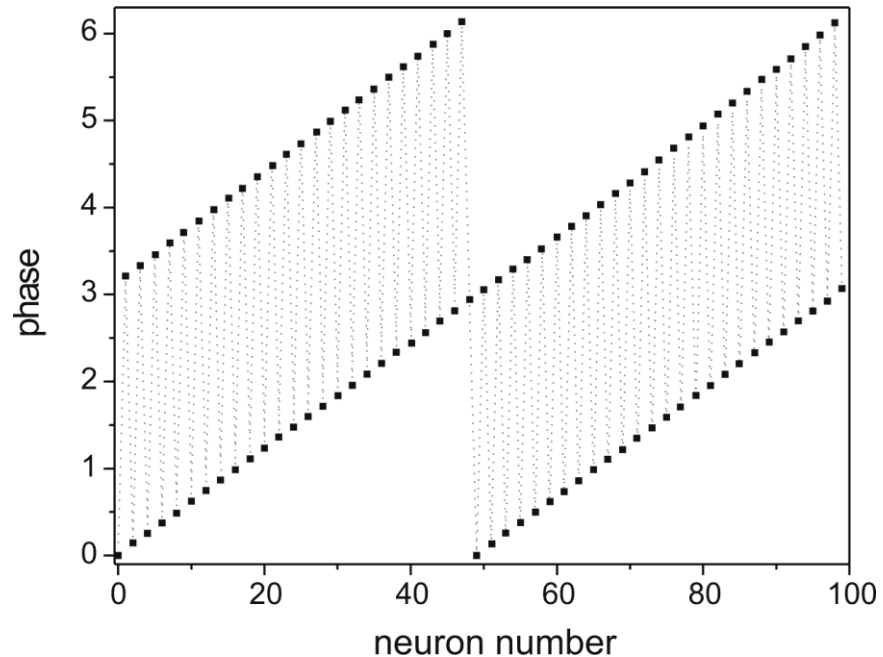
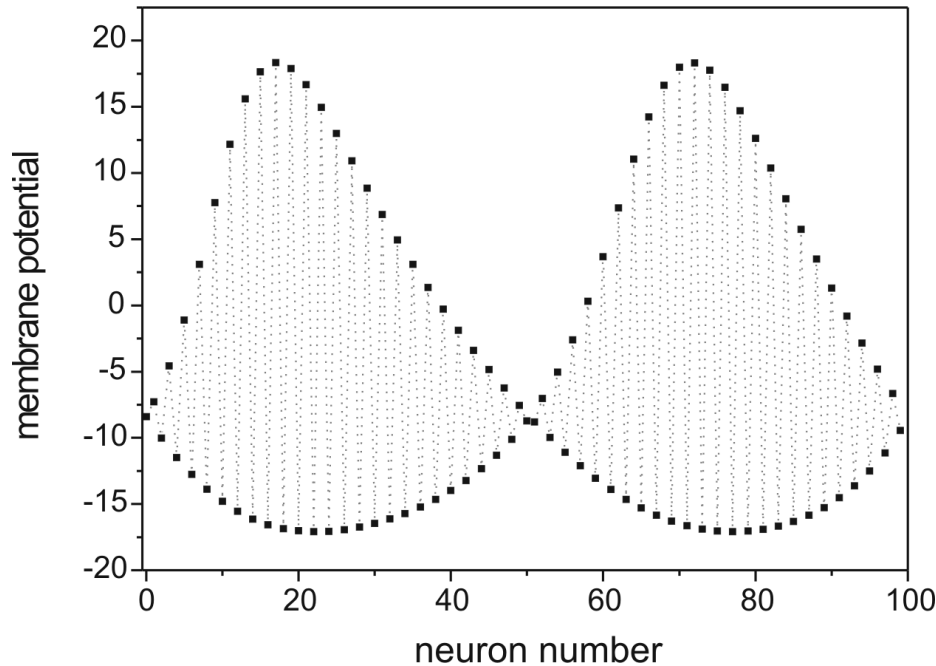


- The ASP structure is such that neurons are anti-phase synchronized in pairs. **Stability of anti-phase synchronization** regime is provided by the functioning of each cell close to the **saddle – node bifurcation and weak coupling** (S. K. Han, C. Kurrer and Y. Kuramoto, *Phys. Review Letters*, V.75, 1995)

- **Small phase shift φ^0** between oscillations of each neuronal pair is strongly required when we speak about traveling structures. Excitation transfer delay determines the ASP velocity in such neural-like media



Phase distribution



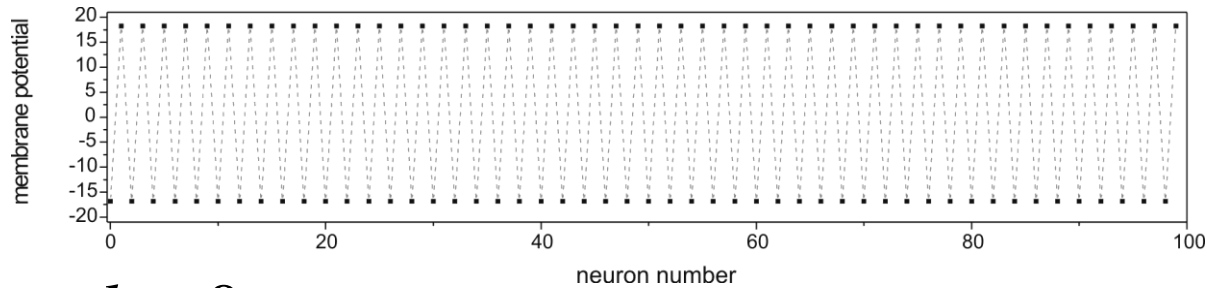
$$\varphi_m = \frac{m - \delta_m}{2} \varphi^0 + \pi \delta_m$$

$$\delta_m = \begin{cases} 0, & \text{if } m \text{ is even} \\ 1, & \text{if } m \text{ is odd} \end{cases}$$

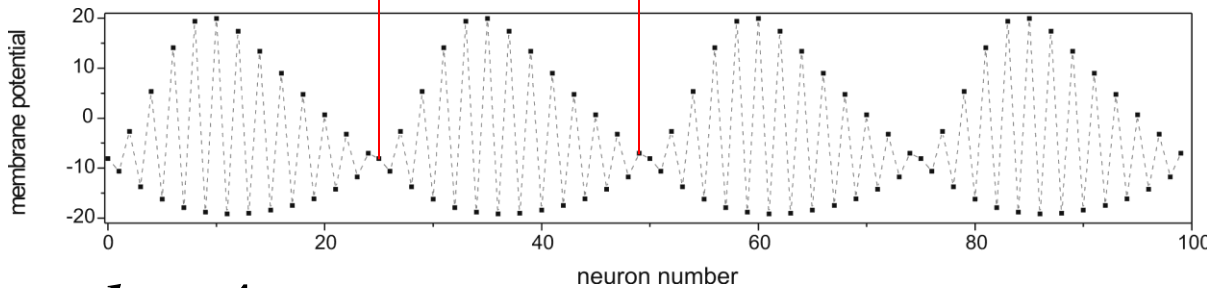
$$m \in [0, N-1]$$

$$\varphi_m = \varphi_{m+N}$$

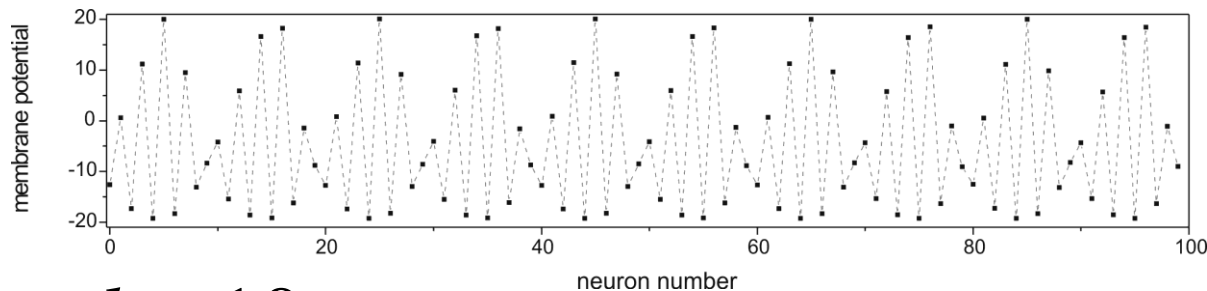
Multistability of the ASP



$k = 0$



$k = 4$



$k = 10$

N is even

$$\lambda_k = \frac{2\pi}{\varphi_k^0} + 2 - \delta_{\lambda_k}$$

$$\varphi_k^0 = \frac{2\pi}{\frac{N}{k} + \delta_{\lambda_k} - 2}$$

Conclusion

1. The special class of spatio – temporal dynamics formed by a chain system modeling electrically coupled neuron-like media has been investigated
2. It has been shown that at the fulfillment of three requirements, the system is capable to produce two types of stable ASP without any external perturbing action. The traveling type has been analyzed more detailed
3. High multistability of the ASP has been indicated. This provides the possibility of basic coding algorithm. Depending on an initial condition pattern, the chain gives an ASP response with the certain spatial period λ_k . Therefore, the chain model introduced in this work can be treated as a basic neural network keeping the memory trace
4. The results, as well as being useful for understanding communication between neurons, are applicable in synchronization problems of nonlinear elements